TABLE VI. Pressure derivatives of polycrystalline elastic moduli at different thermodynamic boundary conditions (at 298°K).

-	and the second s	and the second s		
	Pressure derivatives	dB/dp	dG/dp	dL/dp
5	$(\partial M^s/\partial p)_T$	4.19 ^a 3.70 ^b	1.79ª	6.57ª
	$(\partial M^T / \partial p)_T$	4.23 4.10°	1.79	6.62
	$(\partial M^s/\partial p)_s$	4.16	1.73	6.52

^a These are taken from Table V.

^b This is calculated from the Dugdale-MacDonald relation, i.e., $(\partial B^*/\partial p)_T = 2\gamma_G + 1$, where γ_G is the Grüneisen parameter.

^e This was obtained from the Murnaghan equation of state by a curvefitting procedure using experimental data on compression,

Table VI, two other values of pressure derivative of the bulk modulus have been listed. One is a theoretical value based on the Dugdale–MacDonald relation,²³ and the other is derived from the Murnaghan equation of state²⁴ by a curve-fitting procedure using experimental data on compression.^{9–11} A detailed discussion on these quantities will follow in Sec. 5. It is seen, however, that these values compare reasonably well with the corresponding quantities resulting from the ultrasonicpressure experiments made on both the single-crystal and polycrystalline materials.

4.2. Variation with Temperature

Recalling Eq. (4.1), the temperature derivative of the porosity-sensitive elastic modulus is

$$dM/dT = (dM_0/dT)(1-\alpha\eta) - M_0\alpha(d\eta/dT).$$
 (4.18)

Since $(d\eta/dT)$ is zero, the last term drops out. Thus, dividing the resulting part of Eq. (4.18) by Eq. (4.1), we obtain

$$(1/M) (dM/dT) = (1/M_0) (dM_0/dT).$$
 (4.19)

Equation (4.19) implies that the temperature coefficient of an elastic modulus determined on a porous polycrystalline specimen can be used to estimate the elastic modulus of the nonporous polycrystalline aggregates (as a function of temperature) simply by interpolating the room-temperature modulus of the porous aggregate to that of the nonporous aggregate. The result of such interpolations is given in Table VII. This represents the isotropic elastic parameters of polycrystalline alumina as a function of temperature, which are to be compared with the corresponding single-crystal data.

Figure 3 illustrates the comparison of polycrystalline longitudinal modulus with the corresponding quantity calculated from the single-crystal data using the Voigt-Reuss-Hill (VRH) approximation. The singlecrystal data used here are those of Tefft.⁷ Figure 4 is a similar comparison for the isotropic shear modulus. It is evident that the comparison is good for the shear modulus throughout the temperature range considered. For the longitudinal modulus, we note that there is a significant discrepancy between our data and the VRH modulus calculated from Tefft's single-crystal data (particularly at low temperatures). At present, it is very difficult to see why the longitudinal modulus at temperatures below 100°K calculated from the singlecrystal data decreases with decreasing temperature. Our measurements on polycrystalline specimens indicate exactly the opposite behavior so that the elastic moduli become stiffer as temperature decreases.

5. INTERPRETATION AND DISCUSSION

5.1. Analysis of Temperature Dependence of the Isotropic Elastic Moduli

The total temperature dependence of an elastic modulus can be thought of as consisting of two parts: one, a result of an explicit temperature change, and the other an implicit part resulting from a change in volume with temperature. In other words, the elastic modulus



FIG. 3. A comparison between the measured and calculated isotropic longitudinal modulus as a function of temperature.

²⁸ J. S. Dugdale and D. K. C. MacDonald, Phys. Rev. 89, 832 (1953).

²⁴ F. D. Murnaghan, Proc. Natl. Acad. Sci. **30**, 244 (1944); F. D. Murnaghan, Non-Linear Problems in Mechanics of Continua, Proceedings of the Symposium on Applied Mathematics (American Mathematical Society, Providence, R.I., 1949) Vol. 1.

		Velocities		Elastic moduli				
Temperature (°K)	Density (g/cm ³)	VI	(km/sec)	Um	L ^s (X	10 ¹¹ dyn/cr	<i>B</i> [*] n ²)	$\theta_{D(\text{elastic})}$ (°K)
4.2 77 100 150 200 273 300 400 500 600	3.9924 3.9920 3.9918 3.9910 3.9896 3.9863 3.986 3.978 3.969 3.960	$\begin{array}{c} 10.951\\ 10.947\\ 10.942\\ 10.931\\ 10.919\\ 10.896\\ 10.889\\ 10.844\\ 10.798\\ 10.750\\ \end{array}$	6.448 6.446 6.444 6.437 6.427 6.408 6.398 6.368 6.333 6.296	7.146 7.144 7.141 7.134 7.123 7.102 7.092 7.059 7.021 6.981	$\begin{array}{r} 47.879\\ 47.839\\ 47.793\\ 47.687\\ 47.566\\ 47.326\\ 47.326\\ 47.262\\ 46.778\\ 46.277\\ 45.763\end{array}$	16.599 16.587 16.576 16.537 16.480 16.369 16.369 16.362 16.131 15.918 15.697	25.747 25.727 25.691 25.638 25.593 25.512 25.507 25.270 25.053 24.833	$ \begin{array}{c} 1044(\pm 3) \\ 1043 \\ 1042.6 \\ 1041 \\ 1040 \\ 1036 \\ 1035 \\ 1029 \\ 1023 \\ 1016 \\ \end{array} $
700 800 900 1000 1100 1200 1300	3.949 3.939 3.928 3.918 3.907 3.897 3.886	10.702 10.653 10.602 10.552 10.498 10.444 10.386	6.259 6.218 6.178 6.137 6.096 6.053 6.011	6.941 6.896 6.853 6.809 6.764 6.718 6.667	45.229 44.702 44.152 43.625 43.058 42.507 41.918	$15.470 \\ 15.230 \\ 14.992 \\ 14.756 \\ 14.519 \\ 14.278 \\ 14.041$	24.602 24.396 24.162 23.950 23.700 23.470 23.197	1010 1002 995 988 981 972 965

TABLE VII. Thermo-elastic properties of polycrystalline α -Al₂O₃ (at 1 atm).

M of Born-von Karman-type solids can be treated as a function of volume V (interatomic separation) and temperature T^{25} :

respect to pressure, and rearranging the results, we find $d(\ln M)/dT = -\beta B^{T} [\partial(\ln M)/\partial p]_{T} + [\partial(\ln M)/\partial T]_{V},$ (5.2)

$$M = M(V, T). \tag{5.1}$$

Taking logarithms and differentiating both sides with



FIG. 4. A comparison between the measured and calculated isotropic shear modulus as a function of temperature.

²⁵ D. Lazarus, Phys. Rev. 76, 545 (1949).

where B^T is the isothermal bulk modulus. Hence, from our data on both the pressure and temperature dependences of the isotropic elastic moduli, we should be able to separate out the *changes* due to temperature from those due to volume. Rewriting Eq. (5.2) for the explicit term

$[\partial(\ln M)/\partial T]_{v} = d(\ln M)/dT + \beta B^{T}[\partial(\ln M)/\partial p]_{T}.$

$$(explicit) = (total) + (implicit).$$
 (5.3)

In Table VIII, both the pressure and temperature coefficients of longitudinal, shear, and bulk moduli evaluated at zero-pressure and 298°K are listed. The quantities of our interest $(\partial \ln M/\partial T)_V$, found from these values, are entered in the last column. Any assumption that an elastic modulus is a unique function

TABLE VIII. Pressure and temperature coefficients of the adiabatic elastic moduli of polycrystalline α -Al₂O₃ (at 298°K).

Adiabatic elastic modulus	$d(\ln M)/dT$ (×10 ⁻⁵ /°K)	$ \begin{array}{c} \beta B^{T} \left[\partial (\ln M) / \partial p \right]_{T}^{a} \\ (\times 10^{-5} / ^{\circ} \mathrm{K}) \end{array} $	$\begin{bmatrix} \partial (\ln M) / \partial T \end{bmatrix}_{\mathbf{F}} (\times 10^{-5} / {}^{\circ}\mathrm{K})$
L ^s	-8.23	+5.56	-2.67
G	-9.79	+4.45	-5.34
B*	-6.664	+6.658	-0.006

^a Based on Eq. (5.3), where $\beta = 1.641 \times 10^{-5}$ /°K and $B^T = 25.346 \times 10^{11}$ dyn/cm².